Probability and Random Processes

EES 315

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II. Events-Based Probability Theory

Office Hours:

Check Google Calendar on the course website.

Dr.Prapun's Office:

6th floor of Sirindhralai building, BKD

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5 Foundation of Probability Theory

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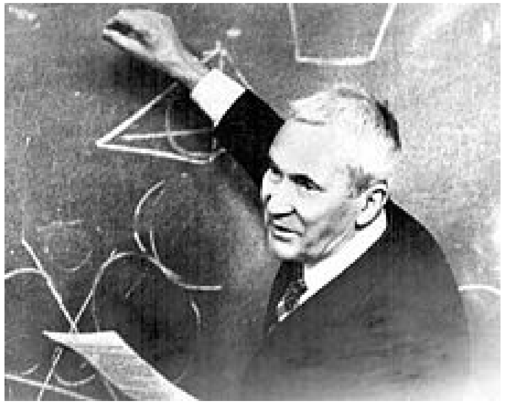
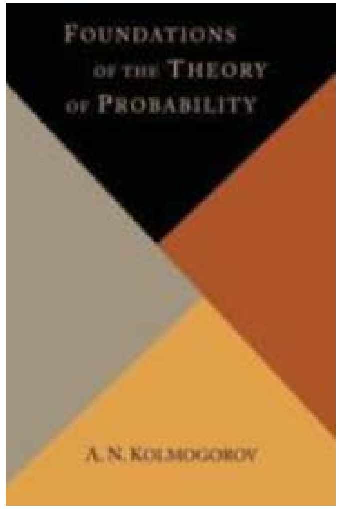
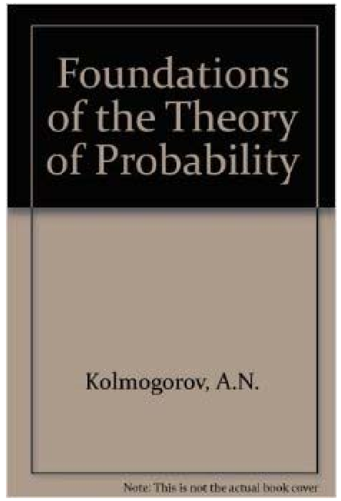
Kolmogorov

- Andrey Nikolaevich Kolmogorov
- Soviet Russian mathematician
- Advanced various scientific fields
  - **probability theory**
  - topology
  - classical mechanics
  - computational complexity.
- 1922: Constructed a **Fourier series** that diverges almost everywhere, gaining international recognition.
- **1933**: Published the book, **Foundations of the Theory of Probability**, laying the modern axiomatic foundations of probability theory and establishing his reputation as the world's leading living expert in this field.

  This book is available at [https://archive.org/details/foundationsofthe00kolm]

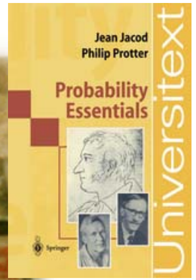






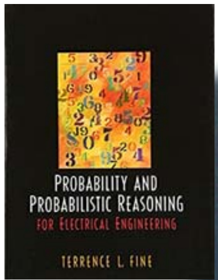
I learned probability theory from

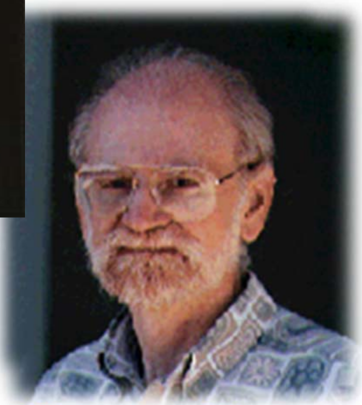
Eugene Dynkin



Philip Protter

Gennady Samorodnitsky

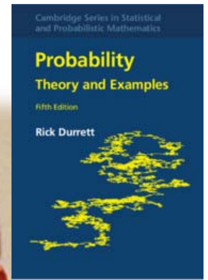




Terrence Fine

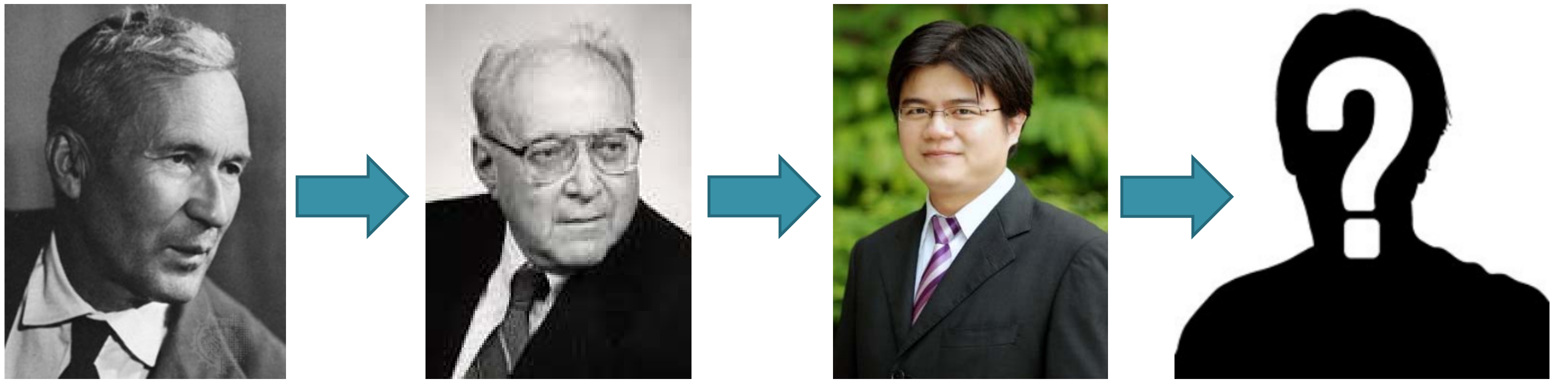
Xing Guo

Toby Berger



Rick Durrett

Not too far from Kolmogorov



You can be

the 4th-generation

probability theorists

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Event-Based Properties

Probability space

- Mathematically, to talk about probability, we refer to **probability space**.
- Probability space has three components
  1. Sample space Ω
  2. Collection of events
     - Example: All subsets of Ω . (Assume Ω is finite.)
  3. Probability Measure
     - A real-valued set function

[Definition 5.1]

Kolmogorov's Axioms for Probability

Abstractly, a **probability measure** is a function that assigns real numbers to events and satisfies the following assumptions:

P1 Nonnegativity: For any event A,

$$P(A) \geq 0.$$

This is called the probability of the event A.

P2 Unit normalization:

$$P(\Omega) = 1$$

P3 Countable Additivity: If $A_1, A_2, \ldots$, is a (countably-infinite) sequence of **disjoint** events, then

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

Additivity

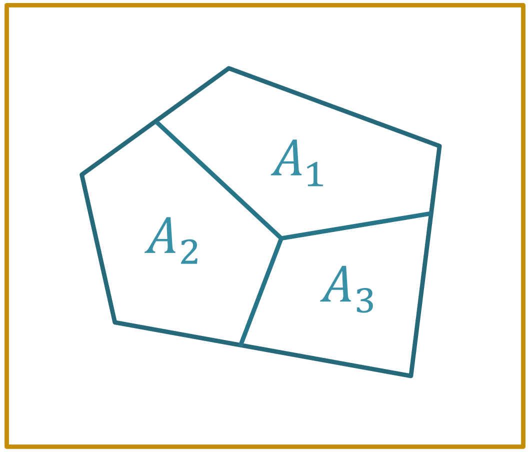
- **Assumption**: $A_1, A_2, \ldots$ are **disjoint** events.

[5.1 P3] - Countable Additivity: $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

[5.4] - Finite Additivity: $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$
  - The formula is quite intuitive when you visualize these events in a Venn diagram and think of their **probabilities** as **areas**.
  - Example:

The "area" of the sample space is 1.

[5.1 P2]



$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3)$

[5.6] Steps to find probability of an event that is defined by outcomes

1. Identify the sample space Ω and the probability $P(\{\omega\})$ for each outcome ω.
2. Identify all the outcomes inside the event under consideration.
3. When the event is countable,
   its probability can be found by adding the probability $P(\{\omega\})$ of the outcomes from the previous step.

$$P(\{a_1, a_2, \dots, a_n\}) = \sum_{i=1}^{n} P(\{a_i\})$$

$$P(\{a_1, a_2, \dots\}) = \sum_{i=1}^{\infty} P(\{a_i\})$$

See [Example 5.7].

More properties

- $P(A) \in [0,1]$.
- Monotonicity: If $A \subset B$, then $P(A) \leq P(B)$.
- If $A \subset B$, then $P(B \setminus \mathrm{A}) = P(B) - P(A)$.
- $P(A^c) = 1 - P(A)$.